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Non-Perturbative Tachyon Potential from the Wilsonian Renormalization Group

James P. O'Dwyer

DAMTP, Centre for Mathematical Sciences
University of Cambridge, Wilberforce Road
Cambridge CB3 0WA, England
j.p.odwyer@damtp.cam.ac.uk

Abstract

The derivative expansion of the Wilsonian renormalization group generates additional terms in the effective β -functions not present in the perturbative approach. Applied to the nonlinear σ model, to lowest order the vanishing of the β -function for the tachyon field generates an equation analogous to that found in open string field theory. Although the nonlinear term depends on the cut-off function, this arbitrariness can be removed by a rescaling of the tachyon field.

1 Introduction

The calculations developed by Friedan and others [1, 2, 3] allow us to express world sheet scale invariance of σ models in terms of conditions satisfied by target space fields. The resulting equations are perturbative in the number of loops computed in the world sheet field theory and are given as an expansion in powers of α' . For example, to lowest order the target space metric must satisfy Einstein's equations, while the tachyon field must solve the Klein-Gordon equation with negative mass-squared. In this note we apply the Wilsonian, or Exact RG (ERG) techniques invented by Wilson [4], Wegner and Houghton [5] and Polchinski [6], and developed by many others more recently (for example see [7, 8, 9, 10, 11, 12, 14] and references therein) to the same model. Earlier applications of the ERG to σ models can be found in [15, 16, 17, 18, 19] and references.

The advantage of these ERG techniques is that there is no reliance on a perturbation expansion, and the resulting conditions for scale invariance might be expected in some sense to go beyond the known perturbative results. The disadvantage is that one always has to make some kind of approximation; the generic effective action will contain all field operators compatible with the symmetry of the problem, and so some kind of truncation is necessary to even begin a calculation. In the next section we will write the effective action as an expansion in world sheet derivatives, and curtail this expansion at the lowest order. This is the local potential approximation. In terms of fields on the target space of the sigma model, we have a tachyon field propagating on fixed, flat geometry. Imposing scale invariance yields equations for the tachyon field, which we will compare to known results from open string field theory [20, 21, 22, 23, 24].

2 The Local Potential Approximation and Tachyon Field Equation

We regularise the theory and introduce a mass-scale Λ by multiplying the propagator (with momentum p) by a cut-off function, $K(\frac{p^2}{\Lambda^2})$, which falls rapidly to zero for large values of the argument. The partition function is:

$$\begin{aligned} Z &= \int \mathcal{D}X e^{-S[X; \Lambda]} \\ S[X; \Lambda] &= \int d^2\sigma \left(-\frac{1}{2} X^i(\sigma) K \left(\frac{-\partial^2}{\Lambda^2} \right)^{-1} \partial^2 X^i(\sigma) \right) + S_{int}[X; \Lambda] \end{aligned} \quad (1)$$

where indices are lowered and raised using δ_{ij} and its inverse. When we change the mass-scale Λ the physics is required to remain the same, and in particular the

partition function must satisfy

$$\Lambda \frac{dZ}{d\Lambda} = 0 \quad (2)$$

so that correlation functions and other physical quantities should not depend on what scale we impose the cut-off. The additional terms in the action needed to impose this condition are contained in the effective interaction lagrangian, S_{int} . Any function of the fields compatible with the symmetries of the problem may be present, and so in order to make progress some kind of truncation is needed. We assume a local expansion in powers of world sheet derivatives:

$$S_{int}[X; \Lambda] = \int d^2\sigma \left(\Lambda^2 T(X; \Lambda) + U_{ij}(X; \Lambda) \partial_\alpha X^i \partial^\alpha X^j + \dots \right) \quad (3)$$

with powers of Λ inserted to make the coefficient functions dimensionless. Further, in the Local Potential Approximation (LPA) the world sheet momenta are assumed to be sufficiently small so that we can neglect the U_{ij} and higher derivative terms in Equation 3.

Writing the action in terms of Fourier-transformed world-sheet fields $\tilde{X}(q)$, the partition function is:

$$Z = \int \mathcal{D}\tilde{X} \exp \left[\int d^2q \left(-\frac{1}{2} \tilde{X}^i(q) \tilde{X}^i(-q) q^2 K \left(\frac{q^2}{\Lambda^2} \right)^{-1} \right) - S_{int}[\tilde{X}; \Lambda] \right] \quad (4)$$

We define dimensionless world sheet momenta $p = \frac{q}{\Lambda}$, and from the definition of the partition function:

$$\Lambda \frac{dZ}{d\Lambda} = \int \mathcal{D}\tilde{X} \left(\int d^2p \left[-\tilde{X}^i(p) \tilde{X}^i(-p) p^4 K(p^2)^{-2} K'(p^2) \right] - \Lambda \frac{\partial S_{int}}{\partial \Lambda} \right) e^{-S[\tilde{X}; \Lambda]} \quad (5)$$

Following Polchinski [6], we now choose the following condition for S_{int} and demonstrate that it is sufficient to satisfy Equation 2:

$$\Lambda \frac{\partial S_{int}}{\partial \Lambda} = - \int d^2p K'(p^2) \left(\frac{\delta S_{int}}{\delta \tilde{X}^i(p)} \frac{\delta S_{int}}{\delta \tilde{X}^i(-p)} - \frac{\delta^2 S_{int}}{\delta \tilde{X}^i(p) \delta \tilde{X}^i(-p)} \right) \quad (6)$$

Upon substitution of Equation 6 into Equation 5 we find:

$$\int d^2p K'(p^2) \int \mathcal{D}\tilde{X} \frac{\delta}{\delta \tilde{X}^i(p)} \left[\left[2\tilde{X}^i(p) p^2 K(p^2)^{-1} + \frac{\delta}{\delta \tilde{X}^i(-p)} \right] e^{-S[\tilde{X}; \Lambda]} \right] = 0 \quad (7)$$

up to an overall infinite constant which can be absorbed into the normalization of Z . Hence 2 is satisfied, as claimed. The identity above is essentially that used in [6], and is an application of Gauss' divergence theorem. (As pointed out in [6], this naive manipulation is justified because there is a cut-off).

We can now translate Equation 6 into conditions on the coefficient fields in Equation 3, and then impose scale invariance. Using the LPA, we retain only $T(X)$ in Equation 3, and assume that it can be expanded as a power series in X . Treating X as effectively constant in σ (see e.g. [14]), the derivation is then fairly lengthy but unambiguous [13, 14]. We find that:

$$0 = \Lambda \frac{\partial T}{\partial \Lambda} = \left[-I_0 \frac{\partial^2 T}{\partial X^i \partial X^i} + K_0 \frac{\partial T}{\partial X^i} \frac{\partial T}{\partial X^i} \right] - 2T \quad (8)$$

where

$$\begin{aligned} K_0 &= -K'(0) \\ I_0 &= -\frac{1}{(2\pi)^2} \int d^2 p \, K'(p^2) = -\frac{1}{4\pi} \int_0^\infty du \, K'(u) = \frac{1}{4\pi} [K(0) - K(\infty)] = \frac{1}{4\pi} \end{aligned} \quad (9)$$

(For the case of a scalar field theory, when there are no indices, these equations reduce to those derived in e.g. [10, 14]). Finally, writing the kinetic term in the action in the usual way for a bosonic string, with the conventional factor of $2\pi\alpha'$ [25, 26], Equation 8 becomes:

$$0 = -\frac{1}{2} \alpha' \partial^2 T + 2\pi\alpha' K_0 (\partial_i T)^2 - 2T \quad (10)$$

In a Lorentzian signature the linearized version of Equation 10 is just Klein-Gordon with a negative mass-squared. The appearance of a quadratic term cannot be deduced perturbatively, but the equation satisfied by T in open string field theory [23, 24] is identical in form. We note that this differs from an earlier result by Tseytlin [19], where the coefficient K_0 was found to be explicitly dependent on the cut-off, Λ . The remaining arbitrariness of our K_0 is an inevitable consequence of the truncation of the derivative expansion. However, so long as it is non-zero the arbitrariness is physically irrelevant; the field can always be rescaled to produce any desired coefficient for the quadratic term.

3 Conclusions

Having derived a tachyon field equation using the Wilsonian RG, there remain some notable unresolved questions. The critical issue is whether K_0 is present in a more complete ERG calculation. Also, truncating the world sheet derivative expansion has truncated the α' expansion. The latter is effectively an expansion in target space derivatives, and it would be useful to understand fully the relation to the world sheet derivative expansion. It would further be interesting to understand the nature of the connection to the boundary SFT used to analyse open string tachyons [21, 23, 24,

31, 32], and to Tseytlin's σ model approach to effective actions [28, 29, 30]. It is likely to be necessary for consistency to consider the higher orders in the derivative expansion, where the massless and massive fields become dynamical and couple to the tachyon. However, the privileged role of the flat metric in this approach may mean that a covariant derivation is not possible.

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